Relations and Functions

Basic Definition of a relation:
A relation is a set of ordered pairs of real numbers. The ***domain*** of a relation is the set of all first coordinates in the relation and the ***range*** of a relation is the set of all second coordinates in the relation.

Basic definition of a function:
A function is a relation such that no two ordered pairs have the same first coordinates and different second coordinates.

Examples:
Determine the domain and the range of the following relations and then determine if each relation is a function:

{(1,3) (2,4) (3,5) (6,7) (8,9)}

domain $= \{1, 2, 3, 6, 8\}$, range $= \{3, 4, 5, 7, 9\}$

It is a function.

{(1,3) (2,3) (3,3) (4,3) (5,3)}

domain $= \{1, 2, 3, 4, 5\}$ range $= \{3\}$

It is a function.

{(1,3) (2,5) (0,1) (-1,8) (0,2)}

domain $= \{1, 2, 0, -1\}$

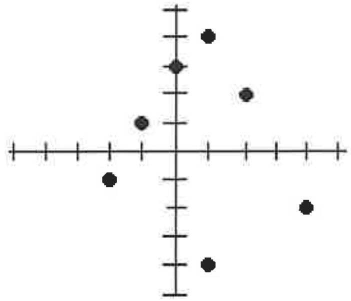
range $= \{3, 5, 1, 8, 2\}$

Not a function.

Examples:

Determine the domain and the range of the following relations and then determine if each relation is a function:

$\{(4,2), (2,2), (-2,-1), (-1,1), (0,3), (1,-4), (1,4)\}$

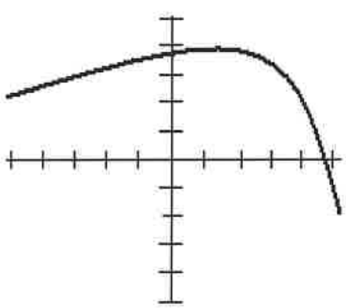


domain $= \{-2, -1, 0, 1, 2, 4\}$

range $= \{-2, 2, -1, 1, 3, -4, 4\}$

Not a function

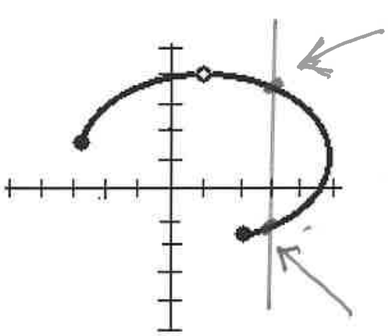
$(1,-4)$, $(1,4)$



domain $= (-5, 6)$

range $= (-2, 4]$

It is a function.



domain $= [-3, 1) \cup (1, 5]$

range $= [-2, 4)$

Not a function

The Vertical Line Test

Examples of equations that are used as rules to define functions.

$y = 2x + 3$

This defines a pair of numbers for every real number x

$\{ (x, 2x+3) : x \in \mathbb{R} \}$

$y = x^2 + 2$

$\{ (x, x^2+2) : x \in \mathbb{R} \}$

$A = \pi r^2$

$\{ (r, \pi r^2) : r \in (0, \infty) \}$

Function Notation :
The equation $y = 2x + 3$ written as $f(x) = 2x + 3$

Suppose $D \subseteq \mathbb{R}$. A real-valued function f is a rule or correspondence such that each real number $x \in D$ is associated with one and only one real number $f(x)$.

$f : D \longrightarrow \mathbb{R}$

Example:
Let $f(x) = \frac{5x^2+2}{x-1}$

What is $f(1)$?

Not defined

What is $f(-2)$?

$\frac{5(4)+2}{-3} = -\frac{22}{3}$

What is $f(a)$?

$\frac{5a^2+2}{a-1}$

What is $f(x+2)$?

$\frac{5(x+2)^2+2}{(x+2)-1}$

What is the domain of f ?

$\{x : x \neq 1\}$

Example:
Let $h(x) = \sqrt{2x+3}$

What is $h(1)$? $\sqrt{5}$

What is $h(-2)$?

$\sqrt{-1}$? Not defined

$= i$?

What is $h(a)$? $\sqrt{2a+3}$

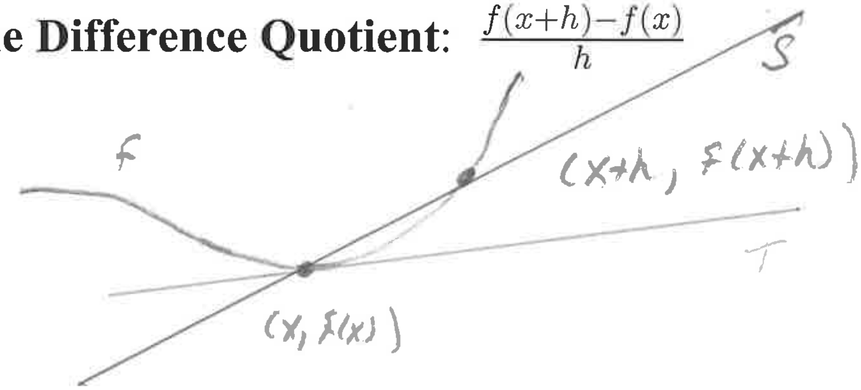
What is $h(x-1)$?

$\sqrt{2(x-1)+3}$

What is the domain of h ? $2x+3 \geq 0$, $x \geq -\frac{3}{2}$

$[-\frac{3}{2}, \infty)$

The Difference Quotient: $\frac{f(x+h)-f(x)}{h}$



If $f(x) = 3x + 2$ then find and simplify the difference quotient

$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)+2-(3x+2)}{h}$$

$$= \frac{3x+3h+2-3x-2}{h} = \frac{3h}{h} = 3, \quad h \neq 0$$

If $s(x) = x^2 + 2x - 1$ then find and simplify the difference quotient

$$\frac{s(x+h)-s(x)}{h} = \frac{(x+h)^2+2(x+h)-1-[x^2+2x-1]}{h}$$

$$= \frac{x^2+2xh+h^2+2x+2h-1-x^2-2x+1}{h}$$

$$= \frac{2xh+h^2+2h}{h} = \frac{\cancel{h}(2x+h+2)}{\cancel{h}}$$

$$= 2x+h+2, \quad h \neq 0$$